GWDG
AG-C
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Exercise 1 / April 20, 2023
Practical Course High-Performance Computing / SoSe 2023
55 Minutes Total

Contents

Task 1: Some QC vector math (5 min)

1. Check that $X|1\rangle = |0\rangle$
2. Check that $H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

Hints

- Remember: $X|0\rangle = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\times 1 + 1\times 0\\1\times 1 + 0\times 0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = |1\rangle$
- Remember:
$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\right] = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

Task 2: Introduction to Qiskit: Entanglement (10 min)

1. Login at `https://jupyter-hpc.gwdg.de/hub/login`
2. Spawn a new server with the option `GWDG HPC with own container`
3. Use `/scratch/users/tmeisel/ISC_23/qiskit.sif` as container path (see figure 1)
4. Upload the two Jupyter notebooks (`.ipynb`) from the `exercises` folder
5. Examine and run`pcpc_qc_exc_1-4.ipynb`
    - Is the probability distribution as you expected?
    - Can you write the corresponding superposition in terms of basis states $|00\rangle$, ...

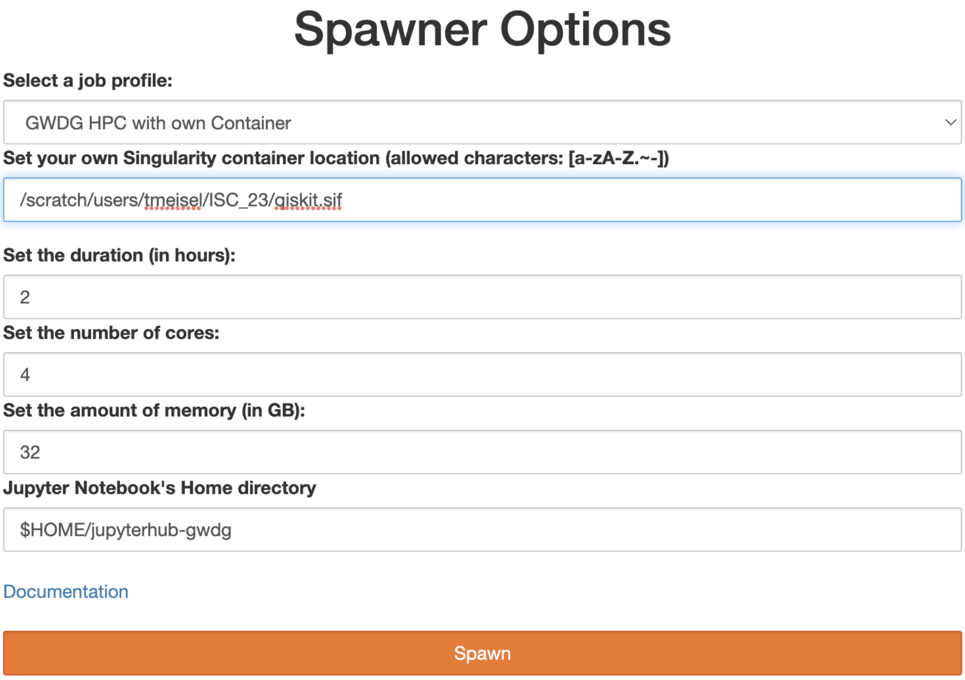
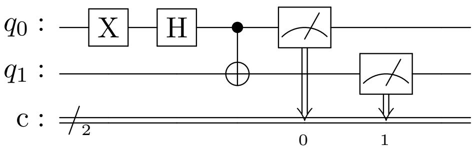


Figure 1: Spawner options.

6. Entangle the two Qubits, but put the control Qubit in $|1\rangle$ state before applying the H gate (see figure 2 for where to make changes)
   - I.e., implement:



   - Think about the result: Is it what you expected? Why or why not?

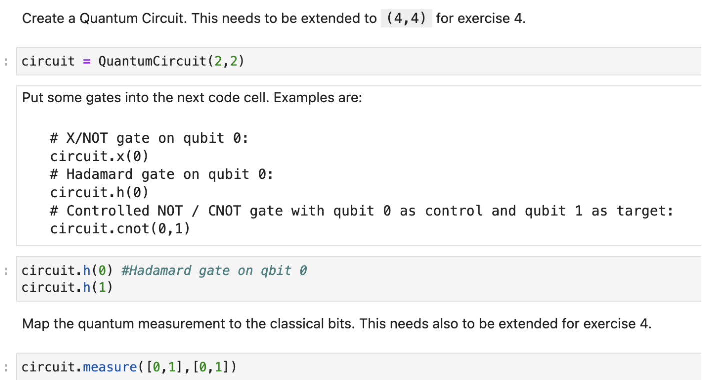
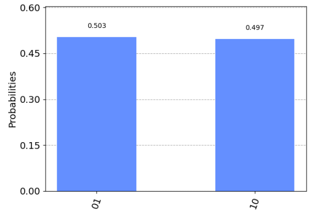


Figure 2: Cells to be changed in exercises 1-4.

Task 3: Modifying entanglement (10 min)

1. There exists an entangled state where 2 Qubits are always in **different** states after measurement:

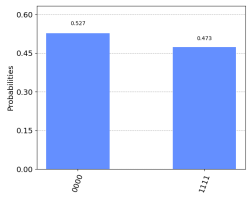


Implement the Quantum circuit producing that state

Optional Task 4: Extending entanglement (10 min)

This is a difficult **additional** task that will support your understanding in the topic.

1. Consider this maximally entangled state of 4 Qubits:



Implement the Quantum circuit producing that state

- For this you need to extend the Quantum circuit to 4 Qubits:
  `circuit = QuantumCircuit(4)`

Hints

- Task 3: You need one additional gate
- Task 4: Start with entangling 2 Qubits
- Task 4: The first three Qubits are now in state $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|011\rangle$
- Task 4: Switch $|011\rangle$ to $|111\rangle$ but **not** $|000\rangle$ to $|100\rangle$. What gate does this?

Task 5: Implementing Deutsch's algorithm (30 min)

1. Open and examine `pcpc_qc_exc_5-7.ipynb`
2. Remember Deutsch's algorithm:
   - Prepare the input (0) Qubit in $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ state
   - Prepare the output (1) Qubit in $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ state
   - Apply the oracle
   - Apply the Hadamard gate to Qubit 0 and 1 each
   - Measure: If $|10\rangle$, the function is constant, else it is balanced

3. Implement pre- and postprocessing as described
4. Implement at least one constant and one balanced oracle
   - Constant oracles are constant $|0\rangle$ and constant $|1\rangle$
   - Balanced oracles are identity and negate

Optional Task 6: Extend Deutsch to Deutsch-Josza (30 min)

This is a difficult **additional** task that will support your understanding in the topic.

1. Use 3 (or more) instead of 1 input Qubits (4+ Qubits in total)
2. Prepare the input and output Qubit states as before
3. You can reuse your constant oracle
4. Implement at least one balanced oracle
   - This should output $|0\rangle$ for half of the possible inputs, $|1\rangle$ for the other half
   - Can you implement a random set of balanced oracles?
5. Implement postprocessing and measuring as before
6. You should measure $|1000\ldots\rangle$ for constant (and something else for balanced)

Optional Task 7: Run your circuit on a real device (X min)

This is a difficult **additional** task that will support your understanding in the topic.

1. The code for this is in `pcpc_qc_exc_5-7.ipynb`
2. You need an account from https://quantum-computing.ibm.com/
3. Retrieve your token from the account and insert in (and uncomment) the according line in the notebook
4. After the first run, the token line can be commented